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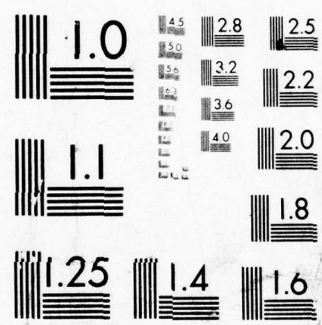

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STRENGTH OF LONG GLASS FIBERS.

BY

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Long silica fibers were drawn and coated in-line with polyethylene. The strength of specimens from 0.05 to 11.96 meters was measured, and the strength distribution of 119 specimens, 1.06 meters long, was also measured. Three methods of estimating the parameters of the Weibull statistical distribution are compared, and the influence of bimodal populations on the shape of an assumed Weibull distribution is demonstrated and applied to the test sample of 119 silica fibers.

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ABSTRACT

The mechanical reliability of long lengths of glass fiber optical waveguides is important to the development of optical communications systems. In the present work, long silica fibers were drawn and coated in-line with polyethylene. The strength of specimens from 0.05 to 11.96 meters was measured, and the strength distribution of 119 specimens, 1.06 meters long was also measured.

Three methods of estimating the parameters of the Weibull statistical distribution are compared, and the influence of bimodal populations on the shape of an assumed Weibull distribution are explored. A technique for extracting a bimodal distribution is demonstrated and applied to the test sample of 119 silica fibers.

CONTENTS

I INTRODUCTION

Optical waveguides using long glass fibers about 100 to 200 μm diameter require reliable and predictable strength characteristics in service. The problem is basically that of predicting the probability of failure, or the minimum extreme strength, of a one kilometer glass fiber from mechanical strength measurements on much shorter samples. The nature of brittle failure, the need to include a significant size parameter in scaling up the test by a factor of 1000 or more, and the need to predict the population behavior from a limited sample all require the application of statistical methods of design reliability.

The purposes of the research carried out under this task were the following: to develop techniques for drawing long silica fibers with in-line polymer coating,to test the mechanical strength of long glass fibers,and to examine the validity of techniques commonly employed to apply the Weibull statistical model to brittle strength data.

The experimental result of this work has already been summarized in the Final Report, "Optical Coupling Techniques, April 1, 1976.(1)

II FIBER FABRICATION AND STRENGTH

1. Drawing Silica Fibers

The final fiber drawing device developed for the production of coated silica fibers is shown schematically in Fig. 1. From the top downwards, the following features were incorporated in the apparatus: (a) Variable speed feed mechanism to lower the preform into the furnace at a constant rate. (b) Graphite sucecptor nitrogen protected induction furnace capable of 2000°C, shown in Fig. 2. Milmaster Model SSE-5R to monitor a preset diameter between 100 to 200 μm within 5%. (d) Polyethylene coating unit consisting of a pressurized, heated annular tank, shown in Fig. 3. (e) A cooling station for air cooling or water mist cooling of the polyethylene coating. (f) Precision machined aluminum winding drum with variable speed drive. (g) Traversing base driven from the winding motor to maintain constant pulling position. This is the same apparatus used in pulling special waveguide shapes for optical coupling structures. (2)

The silica fibers used in this study were drawn from as received 5 mm diameter silica rods.* They were not duplex core cladding fibers nor were optical transmission characteristics controlled or measured. Typical diameters were 125 to 140 μm with a coating thickness of 125 μm.

2. Strength Tests

Attempts to test the strength of fibers several hundred meters long used a drum to drum device described earlier. (2) This approach was abandoned because of occasional random failure of the fiber while resting under tension on the winding drum and the uncertainty and

*Optosil T11 from Amersil, Inc., Hillside, N.J.

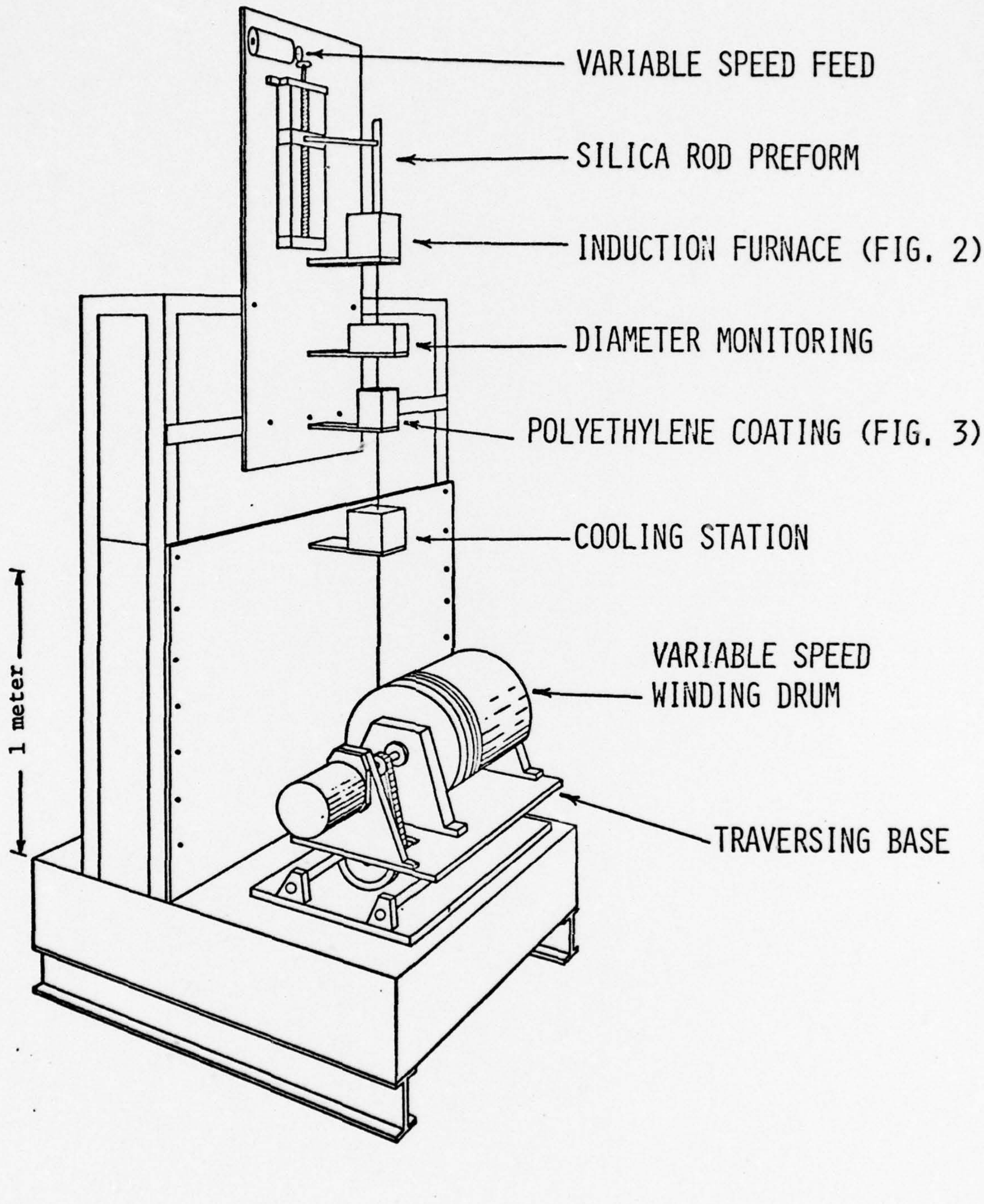


Fig. 1. Fiber Drawing Apparatus. Basic design after Bell Laboratories

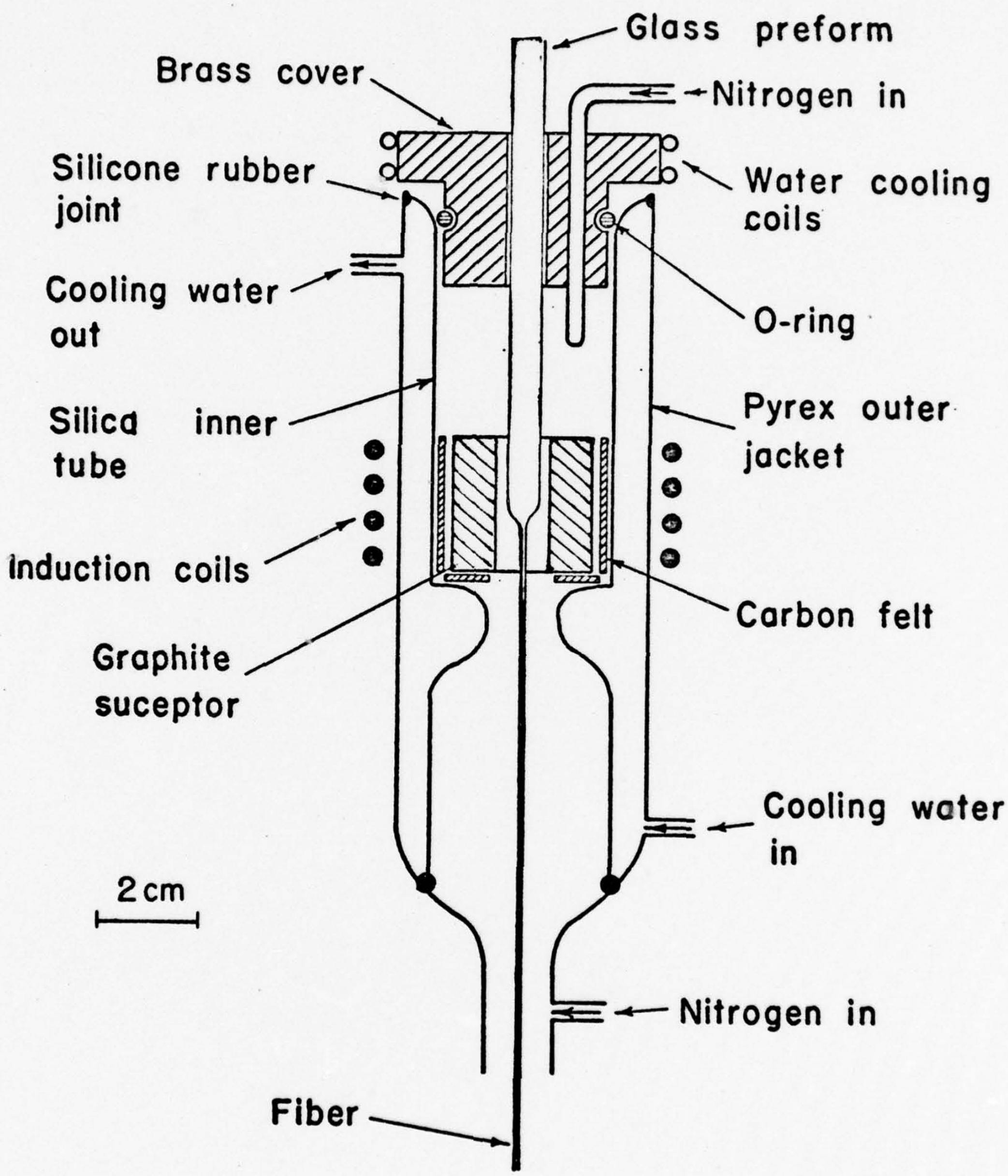


Fig. 2. Induction heater for drawing silica fibers.

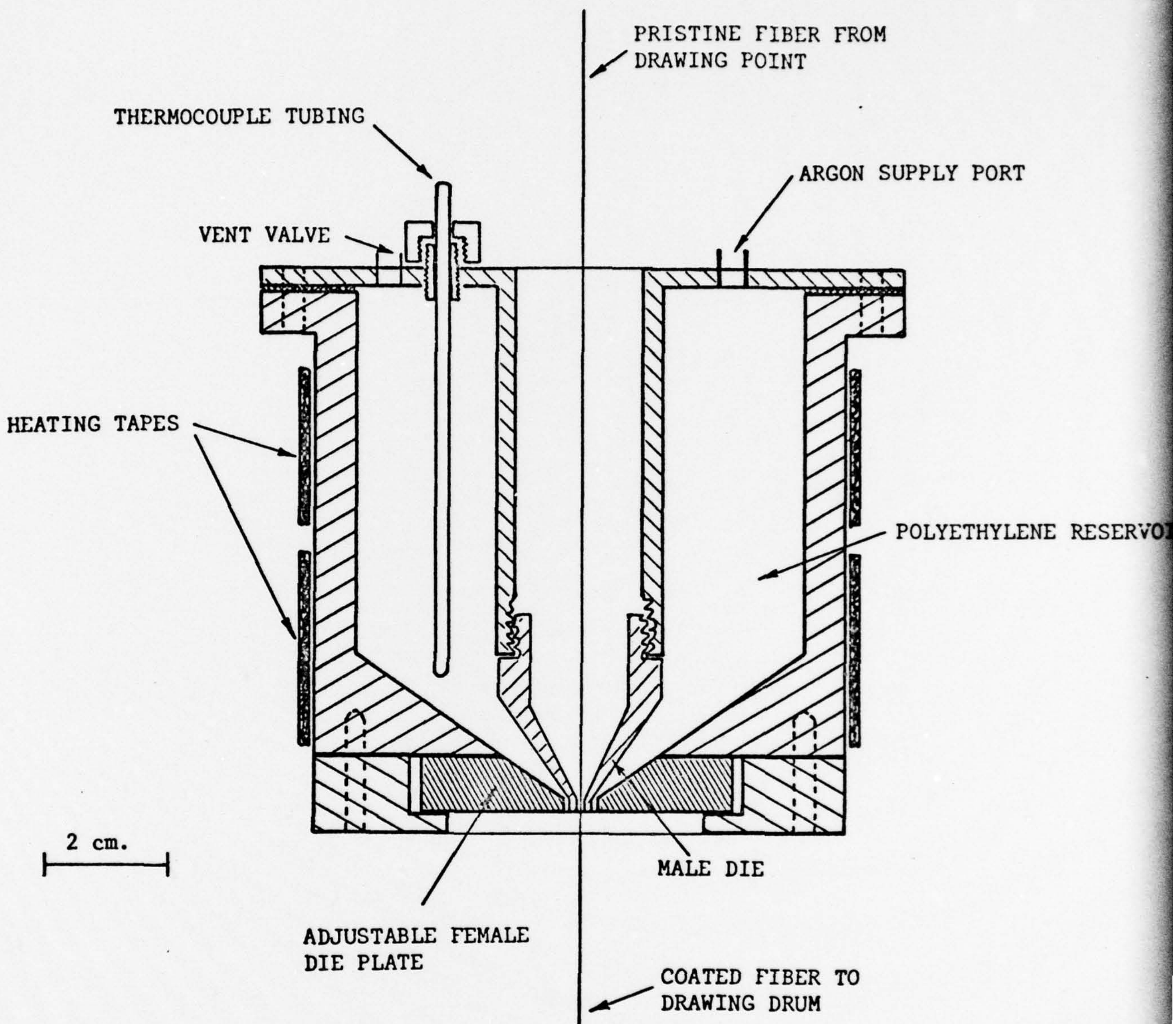


Fig. 3. Polyethylene coating unit.

non-uniformity true stress introduced by curvature on the drums. The best estimate of minimum strength found in fiber lengths up to 300 m was 66.4 MN/m^2 (9500 psi).

Fibers 8 to 12 meters long were tested while suspended vertically in a utility well. Constant loading rate was obtained by running water into a container from a constant head supply. Individual results from these tests are given in Table I.

Fibers 1.4 meter and less in length were tested in an Instron Testing machine at constant strain rate which is equivalent to constant load rate for materials in their elastic range. Table II gives strength results for various size fibers while Table III gives results for a large sample of 119 fibers, 1.06 meters long.

A major experimental difficulty throughout this program was that of obtaining effective mechanical gripping through or on the polyethylene coating. The results reported on Tables I, II and III used grip pads made from polyethylene blocks which were fused to the ends of the fiber with molten polyethylene. This was a very time consuming and tedious procedure. At the high stress levels commonly obtained for short gage length fibers (Table II) many failures occurred by shearing of the coating, and fiber strengths greater than about 2750 MN/m2 (400 ksi) were very difficult to measure. Some later trials using roller grips where the fiber was wrapped several turns around a 3 cm diameter roller surfaced with rubber pads appeared to be a promising technique.

3. Sequential Tests and the Weakest Link Theory

A basic postulate in the theory of brittle fracture is that failure occurs at the largest flaw in the system - the so-called Weakest Link

Table I Tensile Strength of Coated Silica Fibers.

Test No.	Gage Length meters	Fracture Strength MN/m^2
2	11.96	551.6
3	11.96	537.8
4	11.99	448.2
5	8.38	413.7
6	8.28	524.0
7	8.33	475.8
9	8.31	620.5
10	8.41	572.3

Table II   Tensile Strength of Coated Silica Fibers Tested in an Instron Testing Machine

Test No.	Gage Length meters	Fracture Stress MN/m^2
1	0.05	>3102.7
2	"	"
3	"	>2413.
4	"	>2068
5	"	>2758
6	"	>3447
7-17	0.076	>1930
8-18	"	>2102
9-22	"	>1220
10-23	"	>1772
11-24	"	882.6
12	0.102	>2758
13	0.254	>3102
14	"	>2758
15	"	>3102
16	"	>3448
17	"	>3448
18	"	>3448
19	"	>2758
20	"	>2758
21	0.333	>3102
22-15	0.356	>1448
22-16	"	1930.6
23	1.09	1551.4
24	1.14	482.6
25	"	>1379
26	"	2413.2
27	1.17	620.5
28	1.19	827.4
29	1.22	482.6
30	"	>2413
31	1.27	413.7
32-20	1.37	958.4
33-21	"	827.4

Strengths with > symbol indicate failure shear of the coating without fiber fracture.

Table III. Ordered Strength Results for 119 coated silica fiber Specimens 1.06 m long.

Class Number	Strength MN/m^2	Frequency	Cumulative Frequency	Probability (i/n+1)
1	248.2	1	1	.008
2	337.9	1	2	.017
3	482.6	1	3	.025
4	510.2	1	4	.033
5	530.9	1	5	.042
6	537.8	1	6	.050
7	599.9	1	7	.058
8	627.4	1	8	.067
9	634.3	2	10	.083
10	648.1	1	11	.092
11	661.9	1	12	.100
12	696.4	1	13	.108
13	710.2	1	14	.117
14	737.8	1	15	.125
15	744.7	1	16	.133
16	758.5	1	17	.142
17	779.1	1	18	.150
18	786.0	2	20	.167
19	792.9	2	22	.183
20	799.8	2	24	.200
21	806.7	2	26	.217
22	820.5	1	27	.225
23	827.4	1	28	.233
24	834.3	2	30	.250
25	841.2	4	34	.283
26	848.1	1	35	.292
27	855.0	1	36	.300
28	868.8	1	37	.308
29	882.6	2	39	.325
30	903.2	1	40	.333
31	910.1	2	42	.350
32	923.9	2	44	.367
33	937.7	2	46	.383
34	944.6	1	47	.392
35	951.5	2	49	.408
36	958.4	1	50	.417
37	965.3	1	51	.425
38	972.2	1	52	.433
39	979.1	1	53	.442
40	986.0	1	54	.450
41	992.9	4	58	.483
42	999.8	1	59	.492
43	1006.7	1	60	.500
44	1013.6	2	62	.517
45	1027.4	1	63	.525

Table III., (continued)

Class Number	Strength MN/m^2	Frequency	Cumulative Frequency	Probability (i/n+1)
46	1034.2	2	65	.542
47	1048.0	1	66	.550
48	1054.9	2	68	.567
49	1075.6	2	70	.583
50	1089.4	1	71	.592
51	1110.1	1	72	.600
52	1117.0	1	73	.608
53	1123.9	2	75	.625
54	1144.6	2	77	.642
55	1151.5	1	78	.650
56	1158.4	1	79	.658
57	1172.2	1	80	.667
58	1179.0	2	82	.683
59	1192.8	2	84	.700
60	1199.7	1	85	.708
61	1241.1	1	86	.717
62	1254.9	1	87	.725
63	1282.5	1	88	.733
64	1303.2	1	89	.742
65	1316.9	1	90	.750
66	1330.7	1	91	.758
67	1365.2	1	92	.767
68	1420.4	1	93	.775
69	1441.1	1	94	.783
70	1454.8	1	95	.792
71	1475.5	2	97	.808
72	1482.4	2	99	.825
73	1496.2	1	100	.833
74	1537.6	1	101	.842
75	1675.5	1	102	.850
76	1723.7	1	103	.858
77	1751.3	1	104	.867
78	1806.5	1	105	.875
79	1847.9	1	106	.883
80	1972.0	1	107	.892
81	1992.7	1	108	.900
82	2144.3	1	109	.908
83	2323.6	1	110	.917
84	2427.0	1	111	.925
85	2468.4	1	112	.933
86	2640.8	1	113	.942
87	2716.6	1	114	.950
88	2875.2	1	115	.958
89	2978.6	1	116	.967
90	3385.4	1	117	.975
91	3585.4	1	118	.983
92	3771.6	1	119	.992

Theory. In order to test this hypothesis, broken sections of longer fibers were retested. The results are shown in Table IV. In every case, the secondary fractures were at higher stresses than the initial fracture.

Not only does this agree with the weakest link theory, but also shows that the polyethylene coating was effective in preventing externally caused damage to the fibers during handling and testing, and that mechanical properties of the fibers were not degraded by subjecting them to prior stressing. These latter points are important considerations in proof testing for fiber reliability and degradation of fiber strength in service.

4. Statistical Analysis of Strength Results

The data of Table III, 119 specimens, 1.06 meters long, was analyzed in detail to determine if the two-parameter Weibull distribution would adequately model these results and permit valid inferences on the effect of specimen size on strength. In the following sections, the Weibull model and methods of estimating Weibull parameters are briefly reviewed. The effects of variations of Weibull parameters on the shape of calculated distribution functions are shown, and the influence of bimodal distributions on the commonly used linear plotting technique are calculated. The data of Table III is then re-considered as a bimodal distribution, and the size effect on strength is plotted.

Tabel IV Sequential Strength Tests of Silica Fibers

FIRST TEST		SECOND TEST		THIRD TEST	
Gage Length meters	Strength MN/m^2	Gage Length meters	Strength MN/m^2	Gage Length meters	Strength MN/m^2
Test 9, Table I					
8.33	620.5 ---→	1.02	882.6		
		1.02	703.3		
		1.02	675.7 ------→	0.04	855
				0.08	>1710
				0.08	>2758
				0.30	>1517
				0.30	>1237
Test 32-20, Table II					
1.37	958.4 ---→	0.06	>2517		
		1.02	>2034		
		1.22	1172.1		
Test 7, Table I					
8.33	475.8 ----→	0.91	827.4		
		1.07	586.1		
		1.07	599.9		
		1.07	910.1		
		1.07	792.9		
		1.07	861.9		
		1.52	620.5		

III STATISTICS OF THE STRENGTH OF BRITTLE MATERIALS

The mechanical failure of a brittle material is caused by the propagation of a microscopic crack or Griffith flaw. In the case of glass fibers, these flaws are a few micrometers or less in size and occur primarily on the surface of the material. The flaw size, the number of flaws and their frequency of occurrence may be described by statistical distributions. The general problem is one of predicting the minimum extreme value of strength resulting from a sample (flaw) population of a given size. The exact solution for extreme value distributions has been given by Gumbel (3,4) and Epstein (5,6) has published extreme value formulations for a number of different statistical distributions. One type of extreme value model which has wide applicability in failure statistics of many kinds and brittle failure in particular is the Weibull model (7,8).

The Weibull cumulative distribution function (CDF) relating the probability of failure, $F(x)$ to the strength, x is

$$F(x) = 1 - \exp\{-(x/\sigma)^{\lambda}\} \qquad (1)$$

where σ and λ are the scale parameter and the shape parameter respectively.* The corresponding probability density function (PDF) defined as $dF(x)/dx$ is

$$f(x) = (\lambda/\sigma)(x/\sigma)^{\lambda-1} \exp\{-(x/\sigma)^{\lambda}\} \qquad (2)$$

$$0<x,\sigma,\lambda$$

Transposing and taking the logarithm twice transforms Eq. 1 into

$$\ln \ln (1/(1-F(x))) = \lambda \ln x - \lambda \ln\sigma \qquad (3)$$

---

*The notation and nomenclature will follow that of K.V. Bury <u>Statistical Models in Applied Science</u>, Ref. 9, where the derivation and properties of this and other models are given in detail.

The effect of specimen size on observed strength is actually observed as the variation of the extreme value of flaw severity when the flaw sample size is increased, i.e. the number of flaws "sampled" in a given individual strength test. Following Bury (9) the sample size enters the Weibull CDF as

$$F(x) = \exp\{-(L/L_1)(x/\sigma)^{\lambda}\} \tag{4}$$

where L_1 is a unit length (or area or volume) containing N_1 flaws with associated contant σ_1. L is some greater length with the same number of flaws per unit area and $\sigma=\sigma_1/(N_1)^{1/\lambda}$

At constant probability of failure, e.g. 0.5, the relation of size to failure stress is

$$x_1/x_2 = (L_2/L_1)^{1/\lambda} \tag{5}$$

This was originally established by Weibull (7) on empirical grounds and utilized recently by Tarliyal and Kalish (10), Kalish et al.(11), Kurkjian et al. (12), and Mauer et al. (13).

IV ESTIMATION OF THE WEIBULL PARAMETERS σ and λ

1. Linear Plotting

The most common technique for estimating the parameters σ and λ is to plot the observed data in the linear form of Eq. 3. The expected value of F(x) is obtained from the ordered data where

$$F(x) = i/(n+1) \tag{6}$$

in which i is the ith order of failure and n is the total number of specimens in the sample. In addition to initial estimates of σ and λ, this plot also gives information as to the suitability of the proposed model. Substantial systematic curvature in what should be a linear plot indicated the need for a third parameter

in the Weibull function (9) or, as will be shown later, a bimodal or multi-modal population.

2. Maximum Likelihood Method

A second method called Maximum Likelihood (ML) minimizes the variance of the parameters σ and λ, Thoman, Bain and Antle (14) analyzed this technique for the Weibull distribution and published confidence intervals for the parameters and unbiasing factors for the shape parameter as a function of sample (test) size. The Maximum Likelihood value of λ is found by an interative procedure and the expression given by Thoman (Ref. 14, Sec. 5) converges to within $\pm$ 0.001 in about four iterations using starting values from the linear plot of Eq. 3. The ML estimate of σ is then found directly.

3. Non-Linear Least Squares Estimation

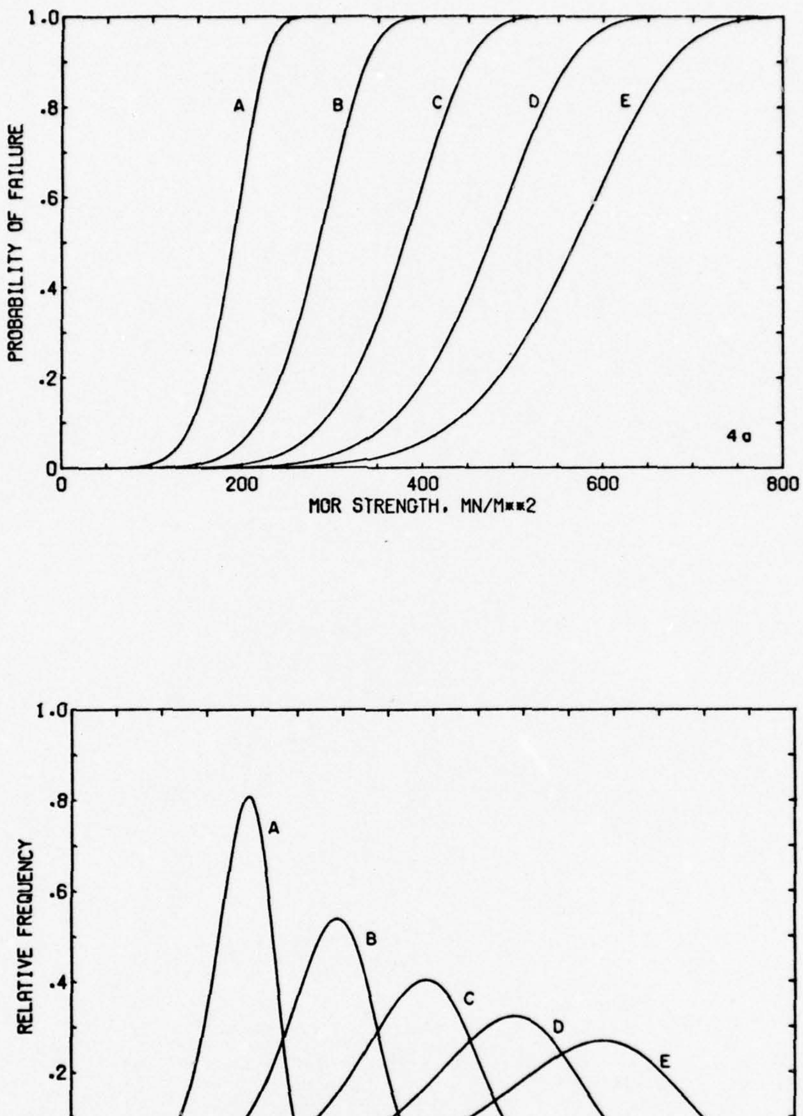
Davies (15) proposed using direct non-linear least squares curve fit to the CDF, Eq. 1 as an improvement in some situations over linear fits to Eq. 3 or maximum likelihood estimates. Non-linear curve fitting programs are not generally available, however, a well documented public computer program called NLWOOD* has been described by Daniel and Wood (16). It is an iterative routine which will evaluate σ and λ directly from Eq. 1. While the program does not minimize the squared residuals, it permits a user selected criteria of relative change in fitted parameters and/or sum of squared residuals as a test of fit. It provides confidence intervals on the parameters, sum of the squared residuals and useful graphical information on the individual residuals. A comparison of the three methods for estimation of Weibull parameters has also been given by Heavens and Murgatroyd (17).

*The source program, FORTRAN listing and User's Manual is available from the SHARE Library (Number 360D-13.6.007), for IBM, and the VIM Library (Number G2-CAL-NLWOOD), for CDC.

V WEIBULL PARAMETERS AND THE RESULTING DISTRIBUTING FUNCTIONS

Theoretical Weibull distributions were generated by assuming values for σ and λ, and a sample size of 60 and calculating the corresponding strength using Equations 1 and 6. Plots were made of Equations 1, 2 and 3 for the assumed parameters to observe the effect of systematic parameter variations on the shapes and locations of the various curves. In addition, the effect of bimodal distributions on the shape of "linear" plots was studied by merging two sets of calculated strength values, reordering, and replotting the merged set as if it was a single unkown sample. The Weibull distributions selected were similar to distributions observed in our laboratory for the strength of 7 and 10mm glass rods in 3-point bending.

Figure 4 shows the effect of varying σ with constant λ, (Table V). This is essentially the size effect for a given population where the smallest specimen size is represented by curve D and the largest size by curve A, nearly 2200 times larger than D. (An extrapolation of 50cm fiber tests to 1km is a size effect of 2000). Although the Weibull parameter λ determines the slope of the linear plot (Fig. 4c) it is clearly incorrect to say that λ determines the "steepness" of the CDF (Fig. 4a) or the dispersion of the PDF (Fig. 4b). These two features which relate to the variability of a set of measurements around the mode or average are a function of both the shape parameter and scale parameters and only in the case where the scale parameter (or the more directly measured, average strength) is the same will the shape parameter alone determine the "steepness" of the CDF.



Figs. 4a, 4b. Calculated Weibull CDF and PDF curves with λ constant and σ increasing left to right. See Table V. This is equivalent to change in specimen size (e.g., length) in a constant population in which A specimens are nearly 2200 times longer than E specimens. The PDF curves are scaled to equal areas.

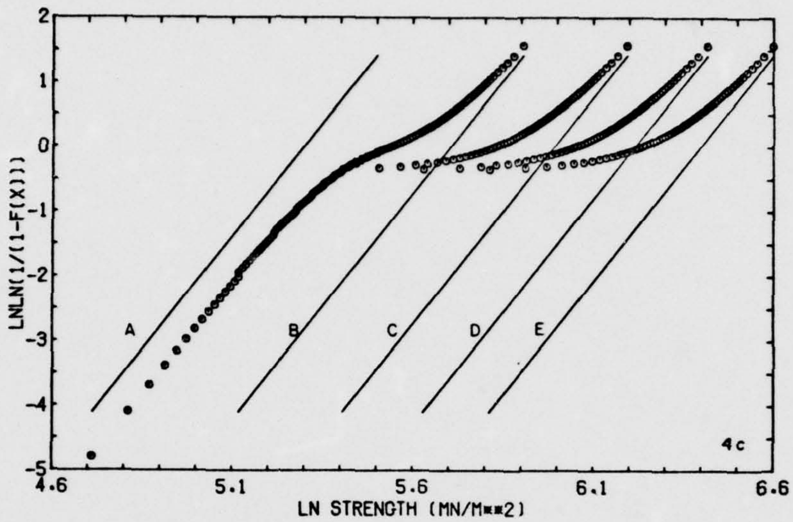


Fig. 4c. Linear forms of the Weibull functions of 4a and 4b. The data points were calculated from two individual functions and then merged, reordered and plotted as if coming from a single sample.

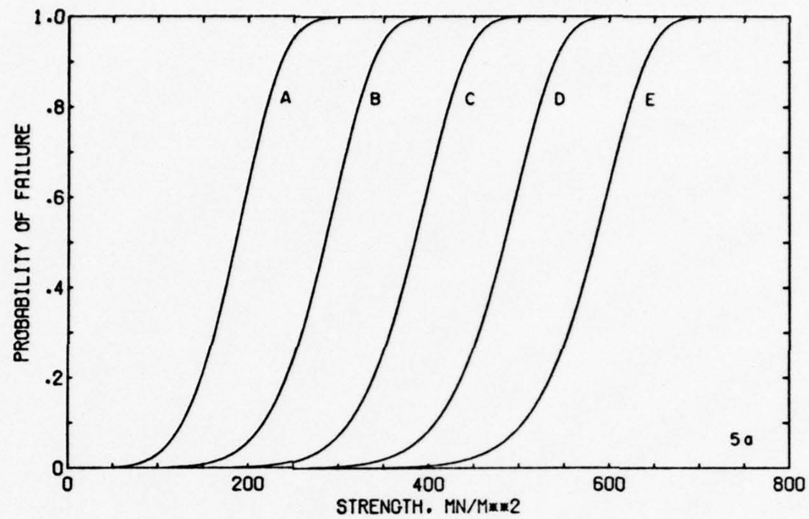
TABLE V

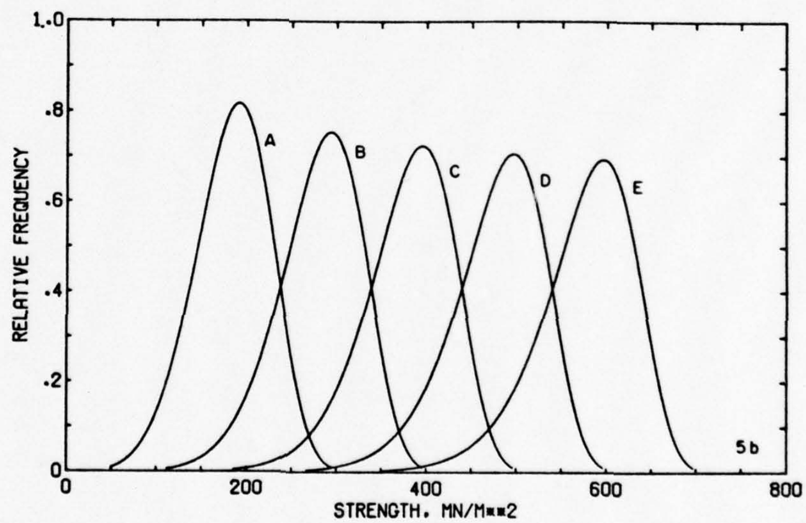
PARAMETERS USED IN CALCULATING WEIBULL FUNCTIONS

		Curve				
		A	B	C	D	E
Fig. 4	σ	200	300	400	500	600
	λ	7.0	7.0	7.0	7.0	7.0
Fig. 5	σ	200	300	400	500	600
	λ	5.0	7.0	9.0	11.0	13.0
Fig. 6	σ	400	400	400	400	500
	λ	7	6	5	4	3

The points in Figure 4c are the merged, calculated values from curves A-B, A-C, A-D, and A-E, i.e. sampling values which might arise if the real populations were an unknown bimodal distribution. As seen in Figure 4c, λ for lower strength distribution (A) would be fairly accurately determined from the slope of the low strength points. However, $\ln\sigma$, which is the value on the abscissa where the linear plot intersects the horizontal line of $\ln \ln (\phi) = 0$, would have substantial error. Virtually no accurate information can be obtained from this plot for the higher distribution of the pair.

Figure 5 is an example where both σ and λ increase (Table V). This produces PDF curves with very similar central portions but different tails skewed to the left. Again errors in estimating σ from the merged samples will arise if the lower strength population is assumed to coincide with the data points. This error occurs in the lower part of Figures 4c and 5c because the order plotting function for each of the true populations is $i/(n+1)$ while for the merged population it was $i/(2n+1)$. As pointed out by Tariyal and Kalish (10), to separate bimodal distributions, one should know the fraction of observations belonging to each distribution. With this information and only the lower part of the strength data, e.g. 30 of the 60 values, the order plotting would be correct and more accurate estimates of σ and λ would result.





Figs. 5a, 5b. Calculated Weibull CDF and PDF curves with increasing λ and σ (Table V).

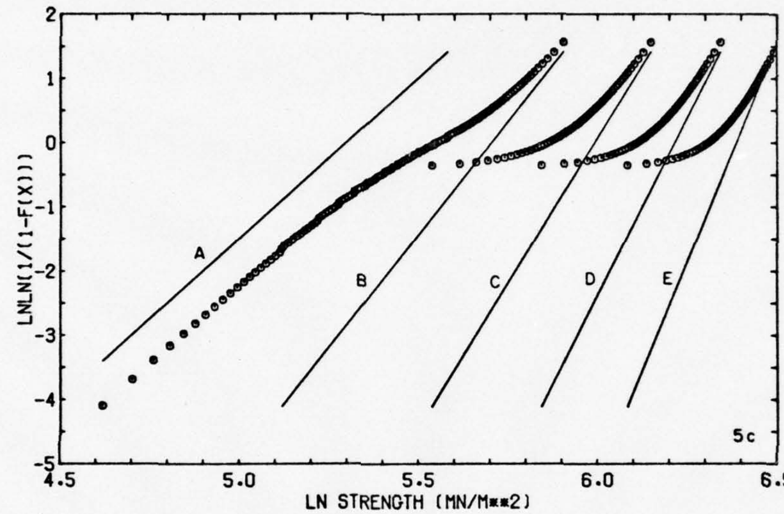
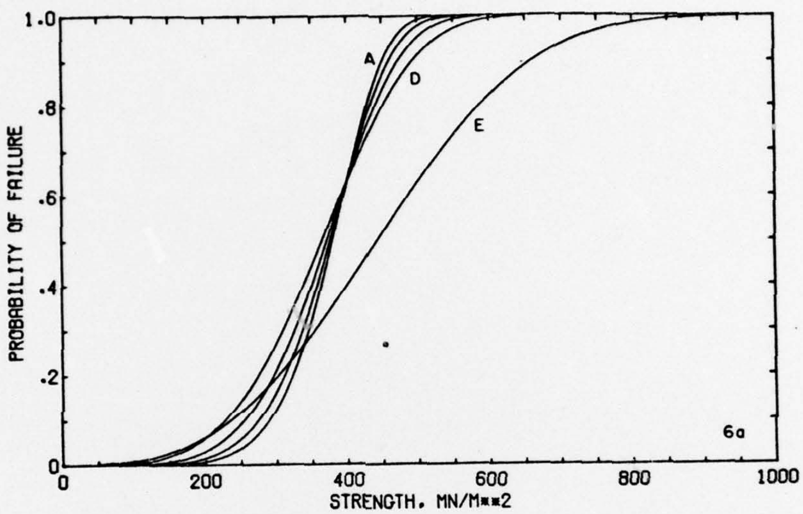
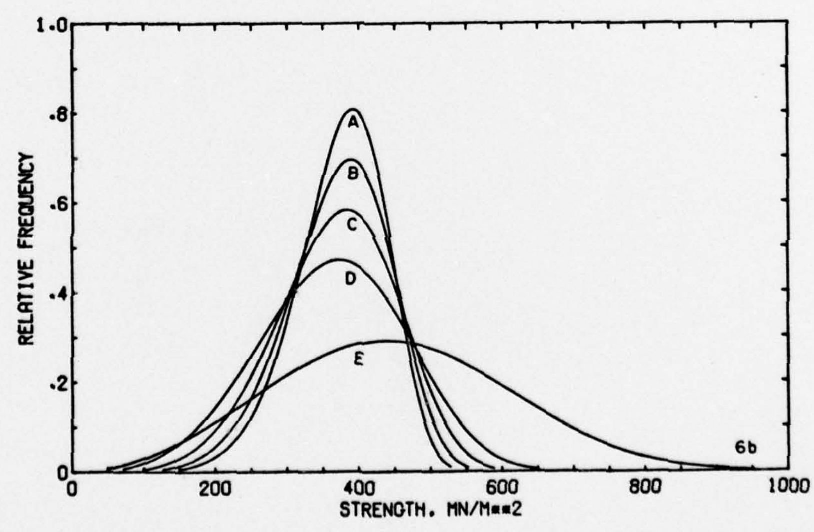


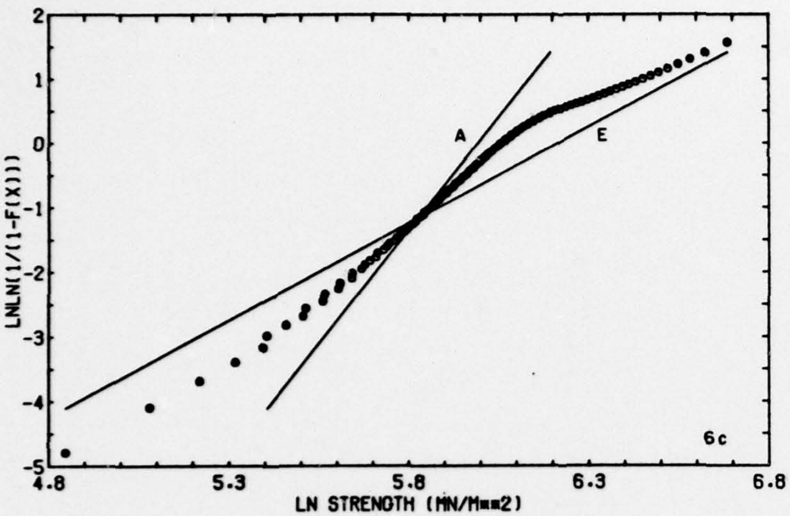
Fig. 5c. Linear forms of the Weibull functions of 5a and 5b. The data points are merged samples as described in Fig. 4c.

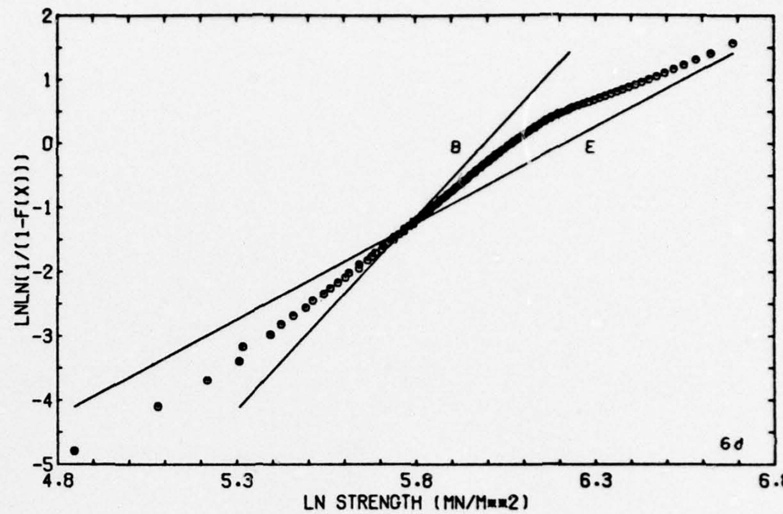
Curves A through D in Figures 6a and 6b have constant σ and decreasing λ. Curve E has higher σ and lower λ (Table V). This set was selected as an example of bimodal distributions with substantial overlap. The shape of the merged points in Figs. 6c, d, and e is similar to that observed for the strength of glass fibers in the present work and by Mauer (18).

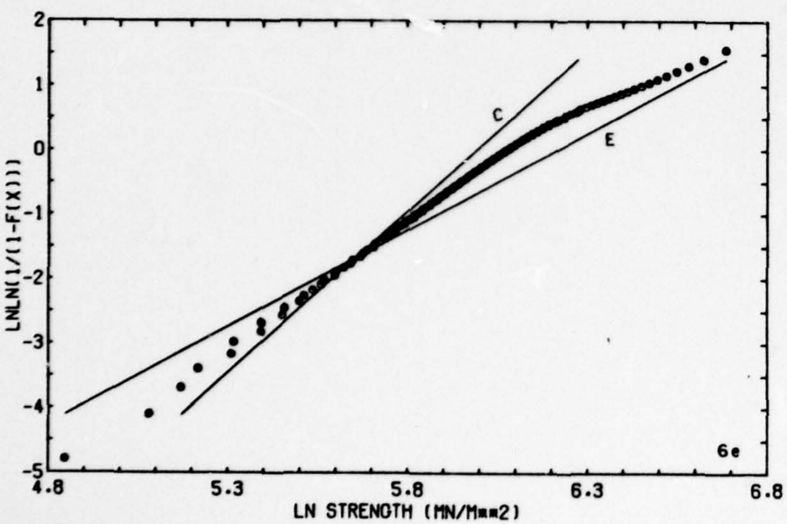




Figs. 6a, 6b. Calculated Weibull CDF and PDF curves. A through D have constant σ and decreasing λ. Curve E has higher σ and lowest λ. (See Table V).







Figs. 6c, 6d, 6e. Linear forms of the Weibull functions of 3a and 3b. The data points are merged as described in 1c.

VI ANALYSIS OF EXPERIMENTAL STRENGTH DATA

Figure 7 shows the direct CDF plot for the 119 specimens 1.06m long listed in Table III. The dashed line is the curve calculated using the ML estimates of the Weibull Parameters, and the solid line was obtained using parameters from the non-linear direct fit. The deviation at the higher strength is very similar to that seen by Davies (15) in silicon nitride. Figure 8 shows the "linear" plot of this data assuming Equations 3 and 6.

One possible correction for the curvature in Figure 8 is the introduction of a location parameter μ in the Weibull CDF as

$$F(x,\mu,\alpha) = 1 - \exp\{-(\frac{x-\mu}{\sigma})^{\lambda}\} \quad (7)$$

This corresponds to a minimum strength, μ, below which failure does not occur. A third parameter was introduced in steps of 35 MN/m^2 from zero to the minimum observed strength of 248 MN/m^2, and the resulting CDF curve calculated at each trial. Although the ML CDF fit (as well as the non-linear direct fit) improves slightly as judged by a reduction in sum of squared residuals, the curvature of the linear plot or the fit at the upper end of the CDF was not significantly improved.

Based on Fig. 6 and other similar trials, the curvature in Fig. 8 was assumed to arise from an overlapping bimodal distribution. The parameters of the two distributions were estimated as follows: Based on straight line portions at the ends of the curve of Fig. 8, the population totals were estimated to be 80 specimens for the lower portion and 39 for the upper portion. These sample sets will be called Set 1 and Set II respectively. Using these

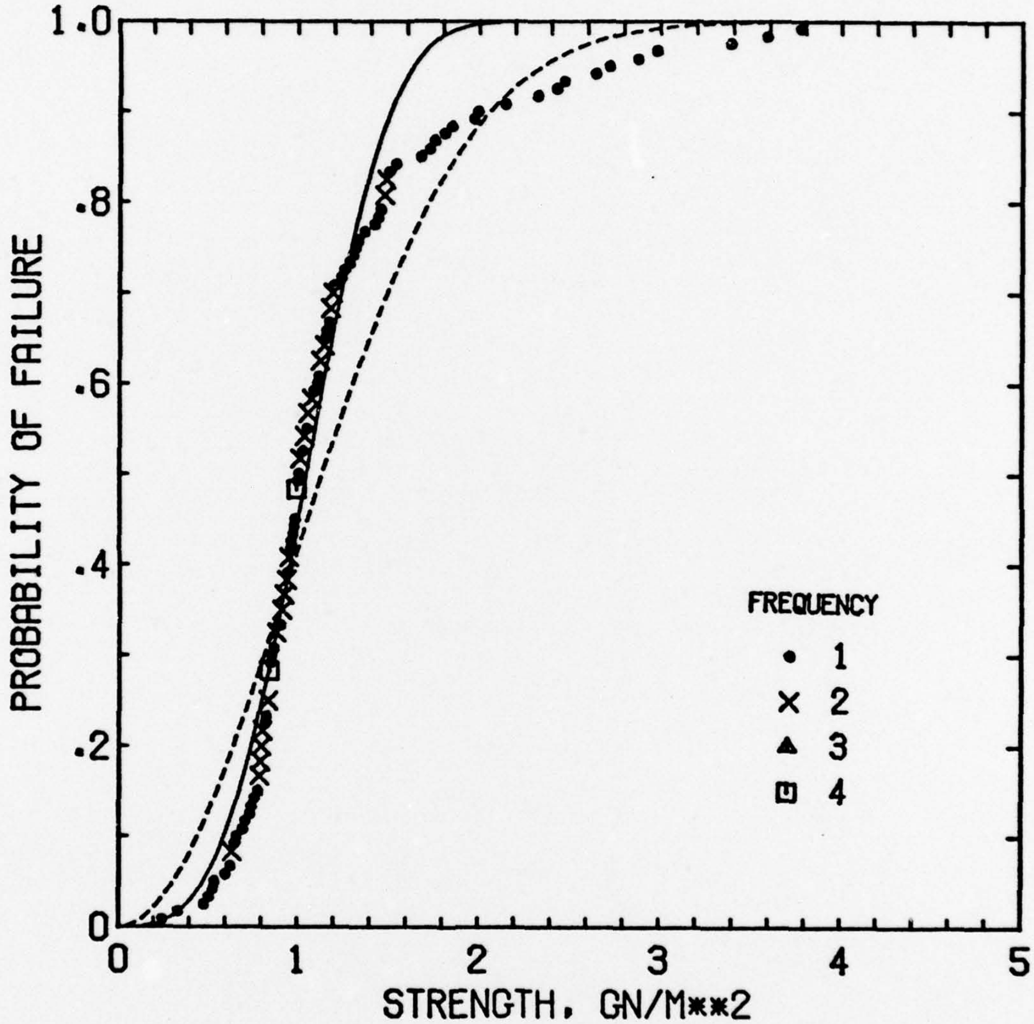


Fig. 7. Direct CDF plot for 119 coated silica glass fibers 106 cm long. The dashed line is the Weibull function calculated from parameters estimated by the Maximum Likelihood Method, the solid line from the non-linear direct fit.

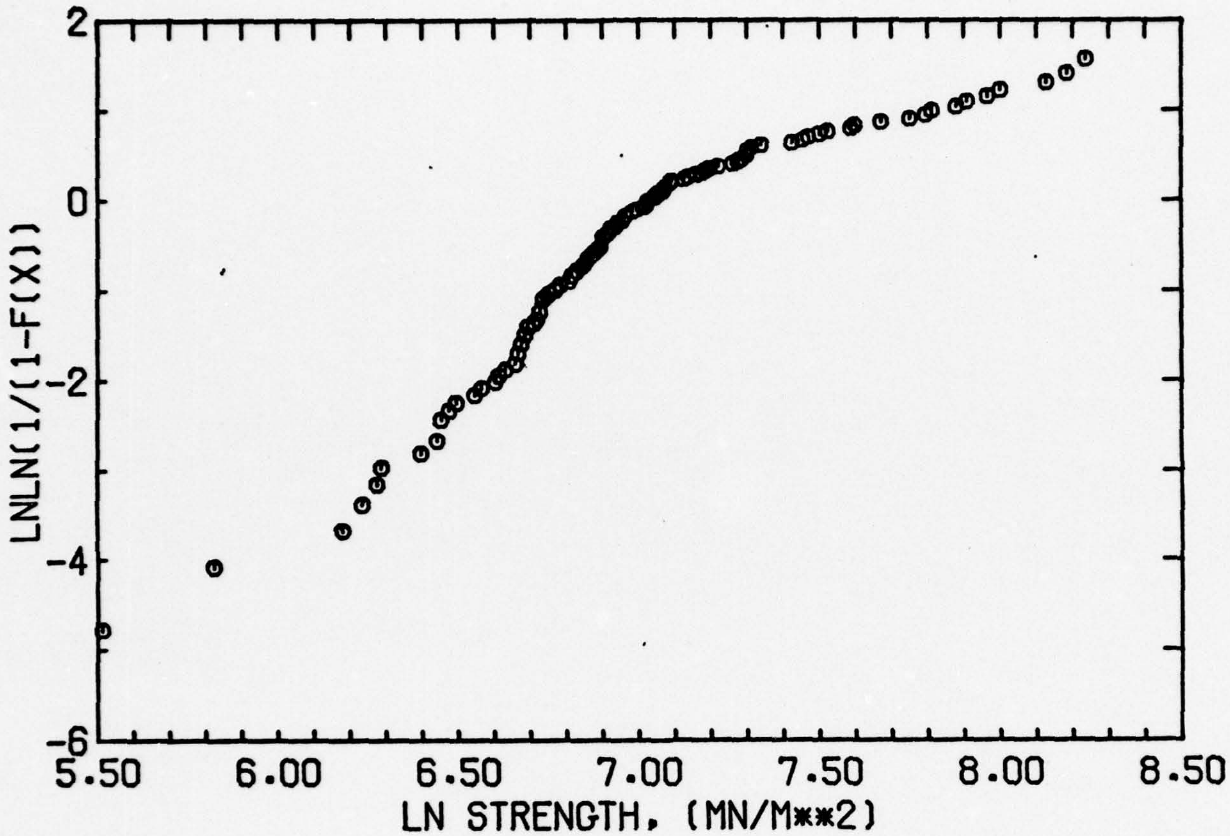


Fig. 8. - Strength of 119 coated silica glass fibers plotted according to Eq. 3.

population sub-sets and assuming the lower 40 and upper 30 data points of the total set to belong to each of the individual distributions alone, the 30 data points were re-ordered against their respective sub-set population. Weibull parameters were estimated using either straight line probability plotting or non-linear curve fitting to this partial data. Maximum likelihood estimates could not be used because the 30 points are not the entire sample.

Two pairs of Weibull parameters σ and λ were estimated from these plots. Theoretical sets of 80 and 39 data points were then calculated using these parameters, and two theoretical sets, i and ii were merged and re-ordered to correspond to the original experimental observation. The ordered positions of individual points belonging to set i and Set ii were determined in the merged population. For example, in the merged 119 theoretical points, the following positions were occupied by points from Set ii: Position No. 1,4,8,14,21,30,39,49,59,67,75,82,87,91,93-to-119. These same positions were then selected from the ordered experimental data and assigned to Set II with the remaining assigned to Set I. Weibull parameters were then recalculated for the experimental data and the results are given in Table VI and Fig. 9. There is a large variation in the parameters estimated by the three different techniques for the full set of 119 specimens. The estimation techniques are much more consistent for the assumed subsets.

One can now calculate the influence of these two distributions on the strength of long fibers, e.g. 1000 m. Using Eq. 4, the failure probability for each distribution at 1km and various stresses

TABLE VI

RESULTS OF VARIOUS METHODS OF WEIBULL PARAMETER ESTIMATION

	Full Data Set, 119				Partial Data, Set I				Partial Data, Set II			
Method	Sample Size	Sigma	Lambda	95% C.L.	Sample Size	Sigma	Lambda	95% C.L.	Sample Size	Sigma	Lambda	95% C.L.
NLLS	119	1170	3.29	3.10 3.49	40 of 80	1005	4.71	4.21 5.21	30 of 39	1916	2.14	1.86 1.97
NLLS	-	-	-	-	80 of 80	1017	5.08	4.94 5.23	39 of 39	1910	2.05	1.94 2.15
LLS	119	1338	2.54	-	80 of 80	1030	4.70	-	39 of 39	1954	2.01	-
ML	119	1359	2.03	1.79 2.26	80 of 80	1029	4.79	4.08 5.45	39 of 39	1942	2.05	1.54 2.50

NLLS = Non Linear Least Squares. LLS = Linear Least Squares. ML = Max. Likelihood.

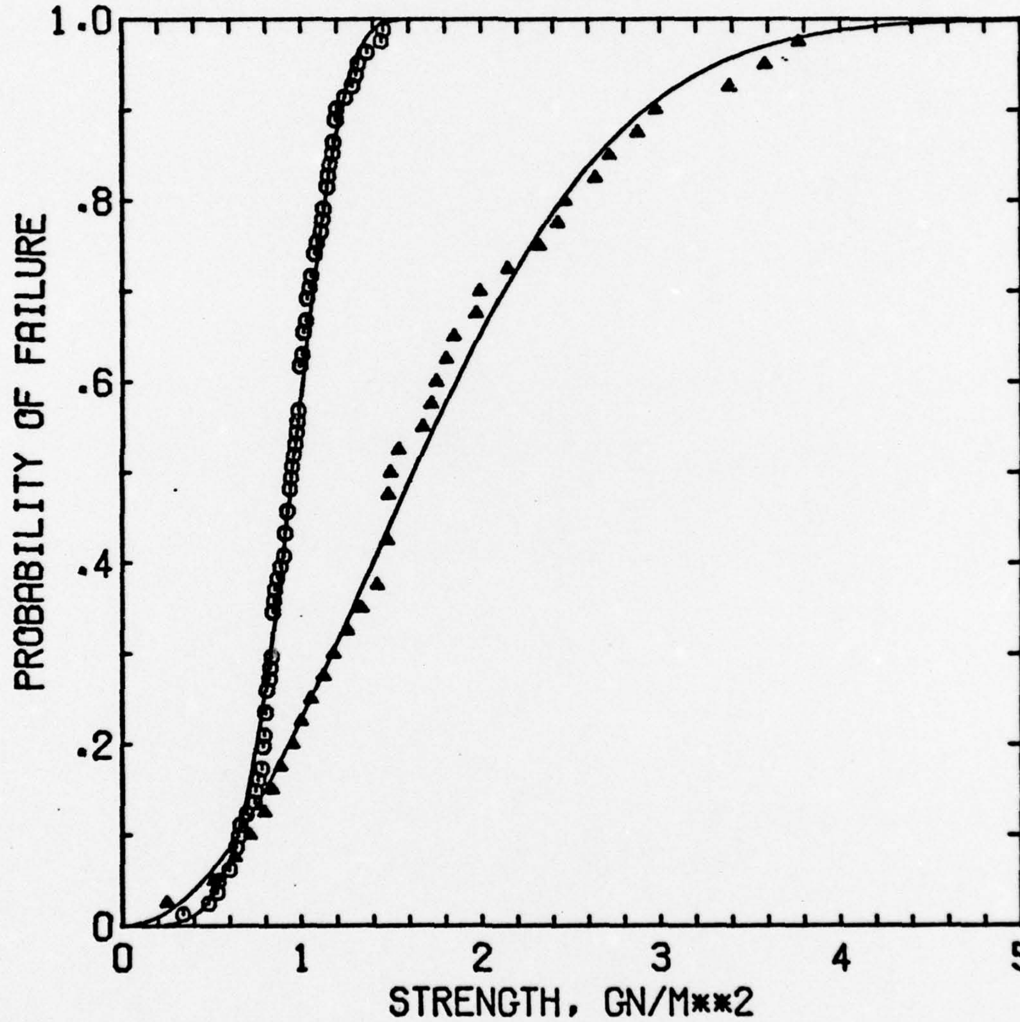


Fig. 9. Weibull CDF curves for assumed bimodal population from the samples of Figs. 7 and 8 (See Table VI).

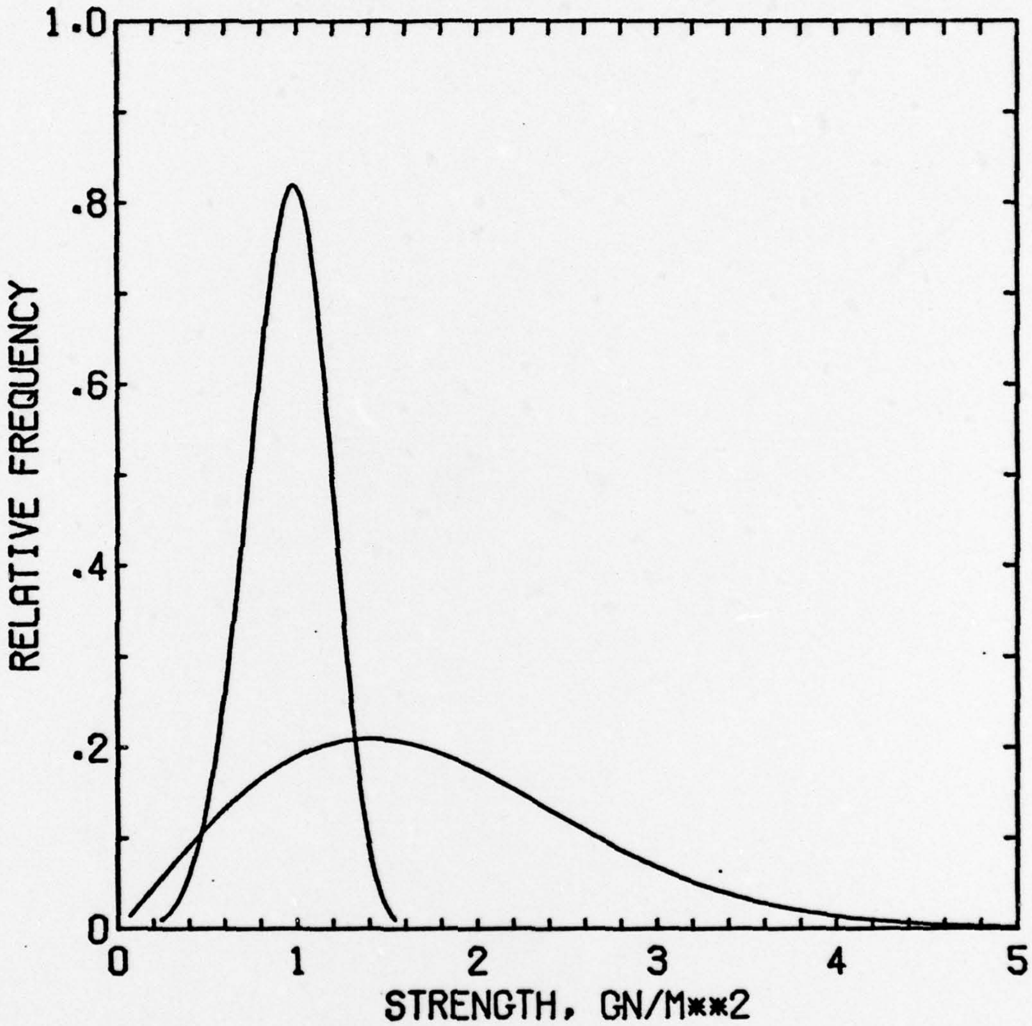


Fig. 10. Weibull PDF curves corresponding to Fig. 9. The curves are scaled to equal areas.

is obtained. The total failure probability is then

$$P_{Total} = 2\ P_1/3 + P_{II}/3$$

assuming the fraction of the populations from each distribution remains 2/3 and 1/3. The results are given in Fig. 11 which shows the calculated size effect on strength using Eq. 1 and the joint probability of two distributions. Data points for the experimental results for fiber lengths from 1.06 to 11.93m are also shown. In calculating the contribution to failure from the two distributions it was found that the tail of the low λ distribution dominated at long lengths. It is interesting that the data from which this distribution was extracted was in the higher strength portion of the original experimental measurement.

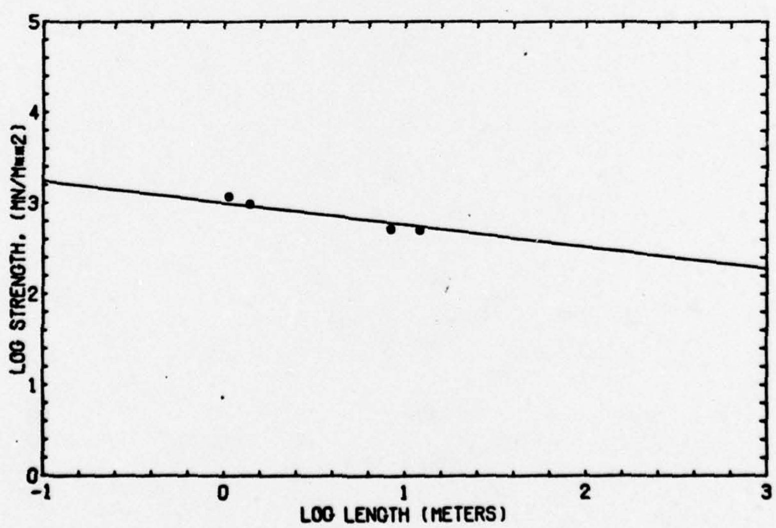


Fig. 11. Average strength of experimental fibers and the calculated 0.5 probability from the joint distribution.

VII CONCLUSIONS

As in any statistical modeling, the foregoing analysis is an attempt to define functions which represent the underlying phenomenon. The fact that one can obtain functions which appear to do this is not a confirmation of the nature of the underlying phenomenon but with hypothesis testing, only places some confidence measurement on the validity of the assumed model.

In mechanical property testing of brittle materials where bimodal distributions are suspected, independent physical confirmation of different failure modes and measurement of the fraction belonging to each distribution should be attempted. Direct fractographic analysis which would indicate different types of flaw origins would be useful. However, it would be very tedious and perhaps impossible for large numbers of high strength fibers. The analytical techniques recently published by Matthews et al. (19) may be useful in defining different independent populations of flaw origins.

Maximum Likelihood method of estimation of Weibull parameters is the most attractive since no ordering of the data is required, the convergence is very rapid, and the form of the expression given by Thoman et al. (14) easily adapted to a computer. Thoman et al. have also provided the analysis of the inferences on the Weibull parameters. However the ML method can not give correct

results if the underlying distribution is not in fact a single Weibull distribution. For this reason, probability plotting should be employed as a check on the assumption of the statistical model and to furnish a first estimate of λ needed for the ML iteration. When the estimates obtained by least squares, non-linear direct fit and Maximum Likelihood are widely different, it is an indication that the assumption of a single modal Weibull distribution is incorrect.

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